

IV

$$x^2 + y^2 = 1 \quad \dots (1)$$

$$4xy = 1 \quad \dots (2)$$

$x > 0, y > 0$ चिह्न \Rightarrow ^{चतुर्थांश} **प्रथम चतुर्थांश**

$$(1) \quad x = \cos \theta, \quad y = \sin \theta \quad (0 < \theta < \frac{\pi}{2}) \text{ चिह्न.}$$

$$(2) \text{ स. } 4 \cos \theta \sin \theta = 1.$$

$$\rightarrow 2 \sin 2\theta = 1 \quad \therefore \sin 2\theta = \frac{1}{2} \quad (0 < 2\theta < \pi)$$

$$\therefore 2\theta = \frac{\pi}{6}, \frac{5}{6}\pi \quad \text{स. } \theta = \frac{\pi}{12}, \frac{5}{12}\pi$$

$$\text{स. } P = \cos \frac{\pi}{12}, \quad Q = \cos \frac{5}{12}\pi$$

$$(2) \quad x^2 + y^2 = 1 \text{ स. } y = \sqrt{1-x^2} \quad 4xy = 1 \text{ स. } y = \frac{1}{4x} \text{ स.}$$

$$S = \int_P^Q (\sqrt{1-x^2} - \frac{1}{4x}) dx = I - J/4$$

$$I = \int_P^Q \sqrt{1-x^2} dx \quad \text{स. } x = \cos \theta \text{ चिह्न. } dx = \frac{d(\cos \theta)}{d\theta} d\theta = -\sin \theta d\theta.$$

$$x: P \rightarrow Q$$

$$\theta: \frac{5\pi}{12} \rightarrow \frac{\pi}{12} \quad \text{स.}$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$I = \int_{\frac{5\pi}{12}}^{\frac{\pi}{12}} \sqrt{1 - \cos^2 \theta} \cdot (-\sin \theta) d\theta = + \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \sin^2 \theta d\theta = + \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{1 - \cos 2\theta}{2} d\theta$$

$$= \left[\frac{-1}{4} \sin 2\theta + \frac{\theta}{2} \right]_{\frac{\pi}{12}}^{\frac{5\pi}{12}} = \frac{1}{8} - \frac{1}{8} + \frac{1}{2} \left(\frac{5}{12} - \frac{1}{12} \right) \pi = \frac{1}{6} \pi.$$

$$\text{स. } \frac{1}{4} J = \int_P^Q \frac{1}{4x} dx = \frac{1}{4} [\log x]_P^Q = \frac{1}{4} (\log Q - \log P) = \frac{1}{4} \log \frac{Q}{P}$$

$$\therefore \text{स. } Q = \cos \frac{\pi}{12} = \sqrt{\frac{1 + \cos \frac{\pi}{6}}{2}} = \frac{\sqrt{1 + \frac{\sqrt{3}}{2}}}{\sqrt{2}} = \frac{\sqrt{2 + \sqrt{3}}}{2} = \frac{\sqrt{(1 + \sqrt{3})^2}}{2\sqrt{2}} = \frac{\sqrt{2}(1 + \sqrt{3})}{4} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$P = \cos \frac{5}{12}\pi = \sqrt{\frac{1 + \cos \frac{5\pi}{6}}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2 - \sqrt{3}}}{2} = \frac{\sqrt{2}\sqrt{(1 - \sqrt{3})^2}}{4} = \frac{(\sqrt{6} - \sqrt{2})}{4}$$

$$\text{स. } \frac{Q}{P} = \frac{\sqrt{6} + \sqrt{2}}{\sqrt{2} - \sqrt{6}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1} = \frac{(\sqrt{3} + 1)^2}{3 - 1} = \frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}.$$

$$\therefore J = \log(2 + \sqrt{3}) \quad \therefore S = \frac{1}{6}\pi - \frac{1}{4} \log(2 + \sqrt{3})$$